Quadratic Functions and Equations in One Variable

1.1 Quadratic Functions and Equations

♥ A quadratic expression in one variable is an expression whereby the highest power for the variable is two.

The general form of a quadratic expression is ax^2+bx+C

Where;

a,b and c are constants and $a \neq 0$, x is variable

EXAMPLE :

Determine whether each of the following expressions is a quadratic expression in one variable. If not, justify your answer.

(a) $2x^2 + 5$

(b) $x^3 - 6$

(c) $3x^2 + 2y + 1$

(d) $\frac{1}{2}m^2$

(a) $2x^2 + 5$ is a quadratic expression in one variable.

(b) $x^3 - 6$ is not a quadratic expression because the highest power of the variable x is 3.

(c) $3x^2 + 2y + 1$ is not a quadratic expression in one variable because there are two variables, x and y.

(d) $\frac{1}{2}m^2$ is a quadratic expression in one variable.

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- ♥ All quadratic functions have the same image for two different objects
- ♥ The type of relation of a quadratic equation function is a many-to-one relation

For a graph of $fx = ax^2+bx+C$

a) There are only two shapes of a graph

b) The value of a determine the shape of the graph

c) The curved shape of the graph of a quadratic function is called a parabola

WHAT IS THE MAXIMUM OR MINIMUM POINT OF A QUADRATIC FUNCTION ?

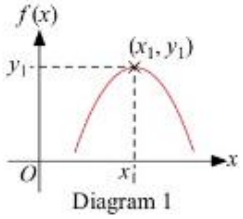

Diagram 1

For the sketch of the graph of a quadratic function with $a < 0$, y_1 is the highest value of y-coordinate and x_1 is the corresponding value for y_1. The point (x_1, y_1) is known as the **maximum point**.

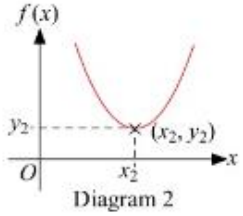

Diagram 2

For the sketch of the graph of a quadratic function with $a > 0$, y_2 is the lowest value of y-coordinate and x_2 is the corresponding value for y_2. The point (x_2, y_2) is known as the **minimum point**.

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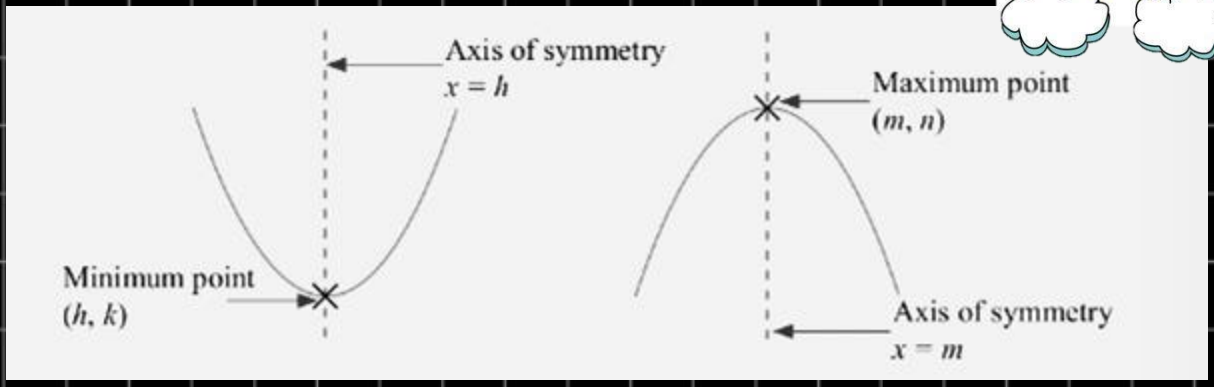
For a quadratic function $fx = ax^2+bx+C$

a) The maximum point is obtained when $a < 0$

b) The minimum point is obtained when $a > 0$

WHAT IS THE AXIS OF SYMMETRY OF THE GRAPH OF A QUADRATIC FUNCTION ?

- The axis of symmetry of the graph of a quadratic function is a straight line that is parallel to the y-axis and divides the graph into two parts of the same size and shape
- The axis of symmetry will pass through the maximum and minimum point of the graph of the function as shown in the diagram below.



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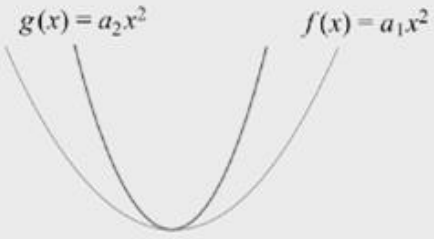
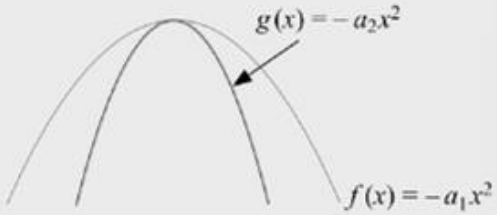
THE VALUE OF A DETERMINES THE SHAPE OF THE GRAPH

For the graph of a quadratic function $f(x) = ax^2 + bx + c$, the smaller the value of a, the wider the curved shape of the graph and vice versa.



$a_1 < a_2$

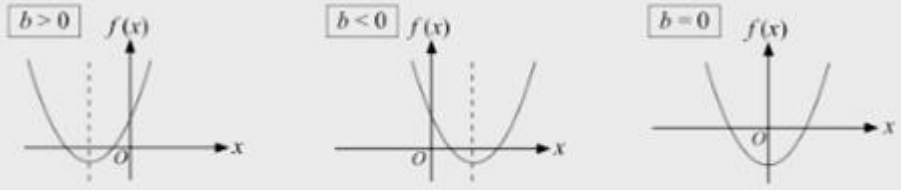


$a_1 < a_2$

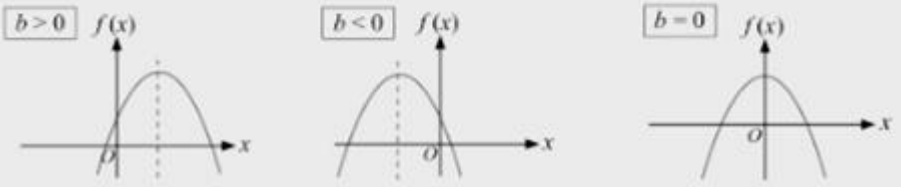
THE VALUE OF B DETERMINES THE POSITION OF THE AXIS OF SYMMETRY

For the graph of a quadratic function $f(x) = ax^2 + bx + c$

if $a > 0$; $b > 0$, then the axis of symmetry lies on the left of the y-axis.
$b < 0$, then the axis of symmetry lies on the right of the y-axis.
$b = 0$, then the axis of symmetry is the y-axis.



if $a < 0$; $b > 0$, then the axis of symmetry lies on the right of the y-axis.
$b < 0$, then the axis of symmetry lies on the left of the y-axis.
$b = 0$, then the axis of symmetry is the y-axis.



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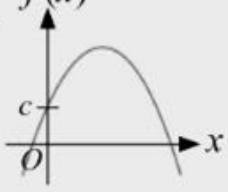
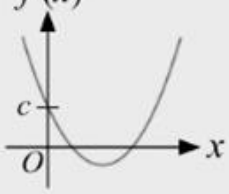
THE VALUE OF C DETERMINES THE POSITION OF THE Y-INTERCEPT

For the graph of a quadratic function $f(x) = ax^2 + bx + c$, the value of c determines the y-intercept of the graph.

$a < 0$



$a > 0$

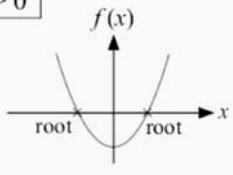


1) There are one or two values of the variable that satisfy a quadratic equation

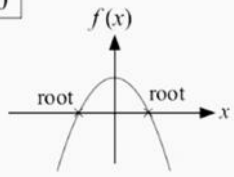
2) The values of the variable that satisfy a quadratic equation are known as the roots of the quadratic equation

The roots of a quadratic equation $ax^2 + bx + c = 0$ are the points of intersection of the graph of the quadratic function $f(x) = ax^2 + bx + c$ and the x-axis, which are also known as the x-intercepts.

$a > 0$



$a < 0$



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Contoh :

(a) $2x^2 - 7x + 3 = 0$

When $x = 1$,

Left: $2x^2 - 7x + 3 = 2(1)^2 - 7(1) + 3$

$= 2 - 7 + 3$

$= -2$ ← not the same

Right: 0

Thus, $x = 1$ is not a root of the equation $2x^2 - 7x + 3 = 0$.

When $x = 3$,

Left: $2x^2 - 7x + 3 = 2(3)^2 - 7(3) + 3$

$= 18 - 21 + 3$

$= 0$ ← same

Right: 0

Thus, $x = 3$ is a root of the equation $2x^2 - 7x + 3 = 0$.

(b) $3x^2 - 7x + 5 = 3$

When $x = 1$,

Left: $3x^2 - 7x + 5 = 3(1)^2 - 7(1) + 5$

$= 3 - 7 + 5$

$= 1$ ← not the same

Right: 3

Thus, $x = 1$ is not a root of the equation $3x^2 - 7x + 5 = 3$.

When $x = \frac{1}{3}$,

Left: $3x^2 - 7x + 5 = 3(\frac{1}{3})^2 - 7(\frac{1}{3}) + 5$

$= \frac{1}{3} - \frac{7}{3} + 5$

$= 3$ ← same

Right: 3

Thus, $x = \frac{1}{3}$ is a root of the equation $3x^2 - 7x + 5 = 3$.

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How do you determine the roots of a quadratic equation by facrtorisation method :

(a) $x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$x = 3$ or $x = 2$

(b) $x^2 + \frac{7}{2}x = 2$

$2x^2 + 7x = 4$

$2x^2 + 7x - 4 = 0$

$(2x - 1)(x + 4) = 0$

$x = \frac{1}{2}$ or $x = -4$

(c) $\frac{x}{2} = \frac{5x - 24}{x - 4}$

$x(x - 4) = 2(5x - 24)$

$x^2 - 4x = 10x - 48$

$x^2 - 14x + 48 = 0$

$(x - 8)(x - 6) = 0$

$x = 8$ or $x = 6$

(d) $(y + 2)(y + 1) = 2(y + 11)$

$y^2 + 3y + 2 = 2y + 22$

$y^2 + y - 20 = 0$

$(y + 5)(y - 4) = 0$

$y = -5$ or $y = 4$

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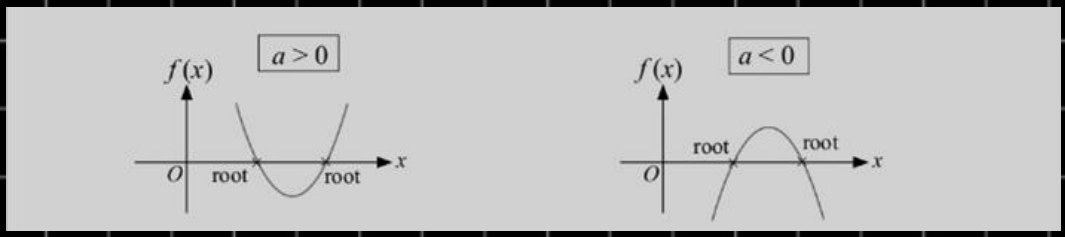
How do you obtained the roots of a quadratic equation using the graphical method ?



HOW DO YOU SKETCH THE GRAPHS OF QUADRATIC FUNCTION?

When sketching the graph of a quadratic function, the following characteristics should be shown on the graph

a) The correct shape of the graph

b) Y-intercept

c) X-intercept or one point that passes through the graph

$f(x) = -x^2 + 2x + 15$

Value of $a = -1 < 0$, shape ∩

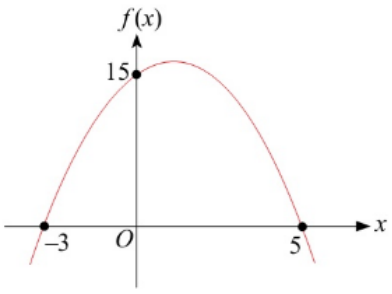
Value of $c = 15$, y-intercept $= 15$

When $f(x) = 0$, $-x^2 + 2x + 15 = 0$

$x^2 - 2x - 15 = 0$

$(x - 5)(x + 3) = 0$

$x = -3$ or $x = 5$



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Conclusion :

Quadratic Expression

- (a) Highest power is 2
- (b) Involves one variable

General form $ax^2 + bx + c$
a, b and c are constants, $a \neq 0$

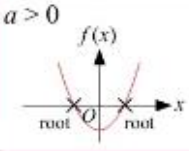
Quadratic Equation

General form
$ax^2 + bx + c = 0$

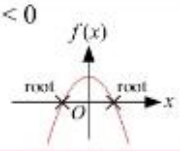
The roots of a quadratic equation are the values of the variable that satisfy the equation

The roots of a quadratic equation can be determined using
(a) factorisation method
(b) graphical method

$a > 0$



$a < 0$



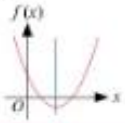
Quadratic Function

General form
$f(x) = ax^2 + bx + c$

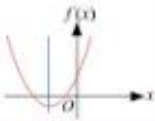
Shape of graph, $a > 0$

Shape of graph, $a < 0$

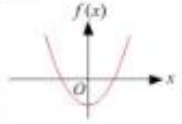
Axis of symmetry
$b < 0$



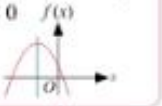
Axis of symmetry
$b > 0$



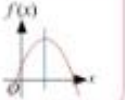
Axis of symmetry
$b = 0$



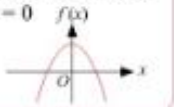
Axis of symmetry
$b < 0$



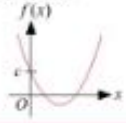
Axis of symmetry
$b > 0$



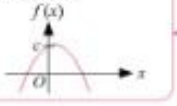
Axis of symmetry
$b = 0$



y-intercept



y-intercept



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